Decoherence from internal degrees of freedom for clusters of mesoparticles: a hierarchy of master equations

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Abstract. A mesoscopic evolution equation for an ensemble of mesoparticles follows from the elimination of internal degrees of freedom. If the system is composed of a hierarchy of scales then the reduction procedure could be worked repeatedly and the characterization of this iterating method is carried out. Namely, a prescription describing a discrete hierarchy of master equations for the density operator is obtained. Decoherence follows from the irreversible coupling between the system, defined by mesoscopic variables, and internal degrees of freedom. We briefly discuss the existence of systems with the same dynamics laws at different scales. We made an explicit calculation for an ensemble of particles with internal harmonic interaction in an external anharmonic field. New conditions related to the semiclassical limit for mesoscopic systems (Wigner-function) are conjectured.

1. Introduction. Reduction technique

The study of models allowing a unified description of microscopic and macroscopic physical systems has a long history. The problem is related to microscopic superposition of states and its non-occurrence at macroscopic scales. Interesting responses and different propositions can be found in [1–17] which are related to coherence destruction by different approaches to the macroscopic level. Many of these theories are related to the original ideas developed by Landau [1] with respect to the high density of states for a macroscopic object, and high sensibility to external perturbation. Thus, any small perturbation produces an undefined (macroscopic) state and then mixture (i.e. decoherence). Nevertheless, it is *naive* to think that the physics between elementary particles and macroscopic objects, for instance macromolecules, can be described only for one mesoscopic theory. This is the case of the DNA-macromolecule which, in a first level, is composed of interacting atoms and finally becomes responsible for transmission of genetic information in a biological level. Other examples are some biological composites such as the hierarchical organization of tendon, bone, mollusk shell, synthetic composites [18, 19] and others.

The scope of this paper is the study of an iterating systematic procedure at different discrete scales of perception or levels. The idea is simple, we start with an ensemble of elementary particles forming clusters around its mass centres, then we eliminate the internal degrees of freedom. In this way the dynamics law for this reduced system is obtained and the procedure could be worked again to the next level (figure 1).

The general assumptions to construct the hierarchy of master equations are as follows.

(i) The system is composed of a hierarchy of scales, or levels, where we can recognize different architectures of clusters.
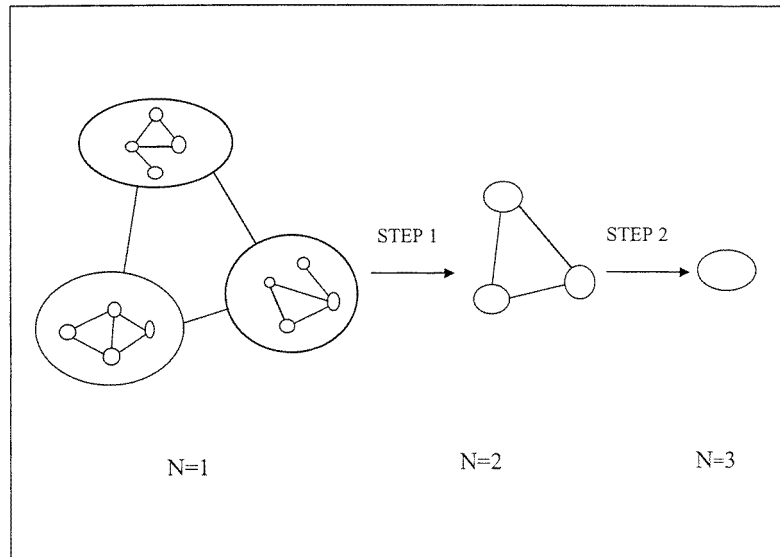
Figure 1. A schematic picture of the reduction procedure (two step). At the first level ($N = 1$), we have different clusters recognition. The full lines represent interactions and, in every step, loss memory effects are assumed.

(ii) There is intercorrelation between levels. Specifically, a given set of clusters make up a cluster in the next scale.

(iii) The dynamical requirement (equations of motion), in a given scale, is dependent on the above one.

Assumptions (i) and (ii) are related to geometrical aspects, and (iii) makes it possible to obtain dynamical laws from more fundamentals scales. In fact, (iii) is related to the belief that phenomenological laws can be explained from fundamental models.

This paper is organized as follows. In this first section, we briefly discuss the standard reduction method applied to an ensemble of generic systems. In section 2 we deal with the iterating method and find the master equation describing the dynamics, at different discrete scales, for interacting mesoparticles. In section 3 some examples are briefly studied and we investigate the question related to the existence of invariant systems under a reduction procedure. In section 4, we use the Wigner function to explore the semiclassical limit for the quantum evolution equation. The mesoscopic term, related to the internal degree of freedom, requires new conditions other than the usual one related to the optical geometric limit. Conclusions and discussions are presented in the final section.

We shall now briefly reviewed the reduction procedure [20–26] which will be used in the next section. Consider the interaction between a system S and other R, with many degrees of freedom, and the evolution equation for the complete system

$$\partial_t \rho = \mathcal{L}\rho. \tag{1}$$

We assume that the Liouville–von Neumann operator is decomposed like

$$\mathcal{L} = \mathcal{L}_S + \mathcal{L}_R + \mathcal{L}_I \tag{2}$$

where $\mathcal{L}_I$ denotes the interaction term assumed time independent. Consider the projector operator P acting on the total density operator ρ (or the space density distribution in the classical case): $P\rho = \rho_R^e \, \text{Tr}_R(\rho)$ where ρ_R^e denotes the equilibrium state of the system R.

In the classical case, the partial trace operation is replaced by an integral over the phase space of R. As usual, the projection operator P satisfies [21]:

$$P\mathcal{L}_R = \mathcal{L}_R P = 0 \qquad P\mathcal{L}_S = \mathcal{L}_S P \qquad P\mathcal{L}_I P = 0. \tag{3}$$

In this way from the evolution equation (1), for the complete system, and projecting on the space spanned by P and $Q = 1 - P$, one obtains the equation for S:

$$\partial_t \rho_S = \mathcal{L}_S \rho_S + \mathrm{Tr}_R \int_{0^-}^t \mathcal{L}_I \mathrm{d}\tau \, e^{Q\mathcal{L}\tau} Q \mathcal{L}_I \rho_R^e \rho_S(t - \tau). \tag{4}$$

To obtain the above equation the usual initial condition $\rho(0) = \rho_R^e \rho_S$ was assumed. Equation (4) is exact and cumbersome because the integral term is dependent on the history of S. From (3) and expanding at second order in the interaction term one obtains the equation

$$\partial_t \rho_S = \mathcal{L}_S \rho_S + \int_{0^-}^t \mathrm{d}\tau \, \langle \mathcal{L}_I \mathcal{L}_I(\tau) \rangle_R \rho_S(t) \tag{5}$$

where $\langle \circ \rangle_R$ denotes the partial trace operation $\mathrm{Tr}_R(\circ \rho_R^e)$, and the symbol '$\circ$' means an element of the space of operators. Assuming the usual memory loss property or Markov approximation:

$$\langle \mathcal{L}_I \mathcal{L}_I(\tau) \rangle_R = \gamma \delta(\tau) \mathcal{L}_{IS}^2 \tag{6}$$

with γ a positive parameter and $\mathcal{L}_{IS}$ an operator acting on ρ_S, then we obtain from (5) and (6), the evolution equation for S

$$\partial_t \rho_S = \mathcal{L}_S \rho_S + \gamma \mathcal{L}_{IS}^2 \rho_S. \tag{7}$$

The explicit verification of the properties: $\rho_S = \rho_S^\dagger$, $\mathrm{Tr}\,\rho_S = 1$ and $\rho_S > 0$ (positivity) must always be carried out. Note that currently an equation like (7) is related to decoherence. Specifically, the reservoir R changes any system pure states to mixed. An approximated case, where $\mathcal{L}_{IS} = \mathcal{L}_S$, with decoherence and without dissipation can be found in [7, 8]. For a criticism to the reduction procedure see for instance [9, 11] where a completely integrable system was considered. Dissipating effects are treated in [4] (nonlinear equation) where other techniques were considered.

2. Reduction procedure. Hierarchy of master equations

In this section we use the reduction procedure sketched in section 1, including a coordinate change to the centre of mass, and we obtain the evolution operator at the next scale or level. Namely, we formalize the procedure $\mathcal{L}^{(n)} \to \mathcal{L}^{(n+1)}$ where n defines the level with $N^{(n)}$ interacting particles forming $N^{(n+1)}$ clusters ($N^{(n)} > N^{(n+1)}$, figure 1) which are recognized by using some physical constraints (assumption (i)). For instance, they could be $N^{(n+1)}$ molecules, composed of atoms, in an electric field or interacting macromolecules composed of molecules. After the cluster recognition, we consider a coordinate change to the centre of mass (assumption (ii)), eliminating the internal degrees of freedom for every cluster (tracing out technique).

Formally at level n, the equation of motion for the density distribution in the classical case, or the density operator in the quantum case, is

$$\partial_t \rho^{(n)} = \mathcal{L}^{(n)} \rho^{(n)} \tag{8}$$

where $\mathcal{L}^{(n)}$ denotes a linear operator constructed, for instance, by elementary Liouvillian (or von Neumann) operators like $\mathcal{L}_f \circ = \{f, \circ\}$ (or $\mathcal{L}_f \circ = (1/i\hbar)[f, \circ]$). Where the symbol $\{\circ, \circ\}$ (or $[\circ, \circ]$) stands for the usual Poisson brackets (commutator).

In (8), the index n becomes related to the discrete scale, and the idea is to obtain the evolution equation, at next scale, by constructing the new operator $\mathcal{L}^{(n+1)}$ from the dynamics at scale n (assumption (iii)). In this way, the reduction procedure has technically two steps.

(a) A coordinate change to the centre of mass of every cluster.

(b) Elimination of internal degrees of freedom by assuming loss-memory-effects (i.e. internal complexity).

Therefore, the reduction $N^{(n)} \rightarrow N^{(n+1)}$ gives an equation similar to (8), where the Liouville–von Neumann operator is determined using steps (a) and (b). To find the new evolution operator, we assume the decomposition:

$$\mathcal{L}^{(n)} = \mathcal{L}_K^{(n)} + \mathcal{L}_V^{(n)}. \tag{9}$$

Namely, a kinetic part depending on momentum and another part depending on position.

First, we explicitly consider the first reduction procedure because it contains all of the basic ingredients for further iterations. Namely, we consider the reduction $\mathcal{L}^{(0)} \rightarrow \mathcal{L}^{(1)}$, where the index $n = 0$ stands for an elementary set of interacting particles forming clusters.

Let $x_{j(\alpha)}$ be the position of the particle j (integer) in the cluster α (integer) where $1 < \alpha < N^{(1)}$. Consider the transformation to the centre of mass y_α, of the cluster α, given by

$$x_{j(\alpha)} = y_\alpha + r_{j(\alpha)} \tag{10}$$

where $r_{j(\alpha)}$ denotes the relative distance with respect y_α. Consider the interacting internal potential U_T,

$$U_T = \sum q_{j(\alpha)}^{k(\beta)} U(x_{j(\alpha)} - x_{k(\beta)}) \tag{11}$$

where q is a coupling parameter and the summation rules on all indices and no self-interactions, or repeated indices, are assumed. Moreover, like $j(\alpha)$, the term $k(\beta)$ denotes the particle k in the cluster at y_β. From (10), the potential transforms like

$$U_T = \sum q_{j(\alpha)}^{k(\beta)} (U(y_\alpha - y_\beta) + (r_{j(\alpha)} - r_{k(\beta)}) U'(y_\alpha - y_\beta)) + F(r) \tag{12}$$

where a first-order multipolar expansion, in the internal coordinates, was assumed and the symbol U' denotes the first derivative. Moreover in (12), $F(r)$ denotes the linear terms depending only on the relative coordinate and related to internal interaction in the cluster. Now we define the coupling parameter, between clusters, Q_α^β and the moment $d_{\alpha,\beta}$ like

$$Q_\alpha^\beta = \sum_{j,k} q_{j(\alpha)}^{k(\beta)} \qquad d_{\alpha,\beta} = \sum_{j,k} q_{j(\alpha)}^{k(\beta)} (r_{j(\alpha)} - r_{k(\beta)}). \tag{13}$$

and from equations (12) and (13) the internal interacting potential becomes

$$U_T = \sum_{\alpha > \beta} Q_\alpha^\beta U(y_\alpha - y_\beta) + d_{\alpha,\beta} U'(y_\alpha - y_\beta) + F(r). \tag{14}$$

Similarly, for an external field acting on every particle and given by

$$V_T = \sum_{\alpha, j} q_{j(\alpha)} V(x_{j(\alpha)}) \tag{15}$$

and defining the new coupling parameters and dipolar distribution by

$$Q_\alpha = \sum_{j(\alpha)} q_{j(\alpha)} \qquad m_\alpha = \sum_{j(\alpha)} q_{j(\alpha)} r_{j(\alpha)} \tag{16}$$

then the potential (15) can be written, at first order in the relative coordinates, as

$$V_T = \sum_\alpha Q_\alpha V(y_\alpha) + m_\alpha V'(y_\alpha) \tag{17}$$

and like (12) the symbol V' denotes the first derivative. Remark that no term like $F(r)$ appears here. From (14), (17), and the transformation for the kinetic term which is form-invariant, the complete Hamiltonian becomes

$$H = H_S + H_R + H_I \tag{18}$$

where the explicit form for the Hamiltonians are

$$H_S = \sum_\alpha \frac{p_\alpha^2}{2\mu_\alpha} + Q_\alpha V(y_\alpha) + \sum_{\alpha > \beta} Q_\alpha^\beta U(y_\alpha - y_\beta) \tag{19}$$

$$H_I = \sum_\alpha V'(y_\alpha) m_\alpha + \sum_{\alpha > \beta} U'(y_\alpha - y_\beta) d_{\alpha,\beta} \tag{20}$$

and H_R is the contribution due only to internal coordinates $(r, \dot{r})$. In equation (19) the term μ_α denotes the total mass of the cluster α. Recall that it was always assumed to be a first multipolar order expansion in the internal coordinate.

In this way, the Hamiltonian (18) has the structure worked in section 1. Since the internal variables $d_{\alpha,\beta}$ and m_α are assumed independent and have the loss memory property then, from section 1, the evolution operator for the ensemble of mesoparticles with position y_α is

$$\mathcal{L}^{(1)} = \mathcal{L}^{(0)}(y, p) + \sum_\alpha \gamma_\alpha^{(1)}(\mathcal{L}_{IV}^{(1)}(y_\alpha))^2 + \sum_{\alpha > \beta} \gamma_{\alpha,\beta}^{(1)}(\mathcal{L}_{IU}^{(1)}(y_\alpha - y_\beta))^2 \tag{21}$$

where we have for every component the expressions:

$$\mathcal{L}^{(0)}(y, p)\circ = \{H_S, \circ\} \tag{22}$$
$$(\mathcal{L}_{IV}^{(1)}(y_\alpha))^2\circ = \{V'(y_\alpha), \{V'(y_\alpha), \circ\}\} \tag{23}$$
$$(\mathcal{L}_{IU}^{(1)}(y_\alpha - y_\beta))^2\circ = \{U'(y_\alpha - y_\beta), \{U'(y_\alpha - y_\beta), \circ\}\} \tag{24}$$

namely a double Poisson brackets, or double commutator in the quantum case.

In (21) the parameters γ are related to white-noise-type correlations between internal variables (dipole moments) for every mesoparticle. This is the Markovian approximation where memory effects are ignored. Expression (21) gives us the evolution operator $\mathcal{L}^{(1)}$ for $N^{(1)}$ mesoparticles of coordinates (y_α, p_α), where internal degrees of freedom were eliminated. We notice that assumption (iii), of section 1, is in accord with our deduction because the evolution operator at $n = 1$, corresponding to $N^{(1)}$ clusters, was deducted from this one at $n = 0$ with $N^{(0)}$ particles. Similar equations for one, or two mesoparticles, were also discussed in [27–29]. The idea to use internal degrees of freedom as an internal environment is also discussed in [30].

The above evolution operator (21) is related to the first elimination of internal degrees. Nevertheless, if we can recognize a second structure of clusters, we can eliminate new internal degrees. Evidently, this will be possible only if the Markovian approximation is valid. Let $\mathcal{L}^{(n)}(x_j, q_j)$ be the evolution operator at scale n with $N^{(n)}$ particles, which includes kinetics and potential terms like (9), and consider a recognition of $N^{(n+1)}$ clusters with centre of masses at y_α where naturally $1 < \alpha < N^{(n+1)}$. Then the first-order multipolar expansion with respect to the relative coordinates (10) is

$$\mathcal{L}^{(n)} = \mathcal{L}^{(n)}(y, Q) + \sum_\alpha \frac{\partial \mathcal{L}^{(n)}(y, Q)}{\partial y_\alpha} d_\alpha + F(r) \tag{25}$$

where the formal derivative stands for the first-order expansion, and d_α are linear functions of the internal degrees r. Moreover, Q denotes some redefined coupling parameters such

as the mass or charge of every mesoparticles. Then we still have a situation similar to that of section 1, and if we assume the loss memory effect then the new evolution operator for the $N^{(n+1)}$ mesoparticles will be

$$\mathcal{L}^{(n+1)} = \mathcal{L}^{(n)}(y, Q) + \sum_{\alpha} \gamma_{\alpha}^{(n)} \left(\frac{\partial \mathcal{L}^{(n)}(y, Q)}{\partial y_{\alpha}} \right)^2. \tag{26}$$

At this point we can state some remarks concerning equation (26). (a) Like the first case ($n = 0$), the parameters γ are related to white-noise type correlation between dipolar moments d of every cluster. So, we assume a hierarchy of timescales where the internal motion is faster than that of every cluster (centre of mass). (b) The formal derivative stands for the first-order multipolar expansion around the centre of mass of every mesoparticle, or cluster. (c) Expression (26) is valid in the quantum or classical case, where $\mathcal{L}$ becomes related to a set of elementary commutators or Poisson brackets. (d) The space structure (lattice, disordered, etc) is contained in $\mathcal{L}^0$. It determines the criterion for the cluster recognition. (f) Remark that the free particles case ($\mathcal{L}_V = \mathcal{L}_U = 0$) is a trivial form-invariant example under the reduction procedure.

3. Examples

We will now briefly examine some examples related to the reduction procedure discussed in section 2. Explicitly, we consider an ensemble of harmonic interacting particles with anharmonic external fields. In fact, the structure of the evolution operator becomes invariant after some reductions (other than some redefined coupling parameters). The search for invariance was also investigated in [3] for a two-parameter model of decoherence, where the equation of motion for the centre of mass is formally identical to the equation for the microscopic constituents.

Consider an ensemble of particles with harmonic interaction, in a nonlinear external field. The internal and external interaction operators are given by

$$\mathcal{L}_U^{(0)} \circ = \sum_{i \neq j} (\tfrac{1}{2}) \{ K_{i,j} (x_i - x_j)^2, \circ \} \tag{27}$$

$$\mathcal{L}_V^{(0)} \circ = \lambda \sum_{j} \{ x_j^3, \circ \} \tag{28}$$

where $K_{i,j}$ are positive constants and λ is a coupling parameter. The expansion around the $N^{(1)}$ centre of mass gives

$$\mathcal{L}_U^{(0)} \circ = \sum_{\alpha \neq \beta} (\tfrac{1}{2}) \{ K'_{\alpha,\beta} (y_{j(\alpha)} - y_{k(\beta)})^2, \circ \}$$

$$+ \sum_{\alpha \neq \beta} \sum_{j,k} \{ K_{j(\alpha),k(\beta)} (y_\alpha - y_\beta)(r_{j(\alpha)} - r_{k(\beta)}), \circ \} + F(r) \tag{29}$$

and for the external component

$$\mathcal{L}_V^{(0)} \circ = \lambda' \sum_{\alpha} \{ y_\alpha^3, \circ \} + 3\lambda \sum_{\alpha,j} \{ y_\alpha^2 r_{j(\alpha)}, \circ \} \tag{30}$$

where K' and λ' are redefined coupling parameters. The reduction procedure (section 2) leads to the evolution operator

$$\mathcal{L}^{(1)} \circ = \sum_{\alpha} \left\{ \frac{p_\alpha^2}{2\mu_\alpha} + \lambda' y_\alpha^3, \circ \right\} + \sum_{\alpha,\beta} K'_{\alpha,\beta} \{ (y_\alpha - y_\beta)^2, \circ \}$$

$$+ \sum_{\alpha,\beta} \gamma_{\alpha,\beta}^K \{(y_\alpha - y_\beta), \{(y_\alpha - y_\beta), \circ\}\} + \sum_\alpha \gamma_\alpha^\lambda \{y_\alpha^2, \{y_\alpha^2, \circ\}\} \tag{31}$$

which gives us the evolution equation for the set of mesoparticles. At this point we have an interesting result, a second reduction process, makes form-invariant the (internal) evolution operator. In fact, the only changes are related to the redefinition of the coupling parameters and mass. The same is true for the external anharmonic term in (31), which becomes invariant after three reduction process. This is a general property related to the formal first derivative in expression (26) for the evolution operator. For systems interacting algebraically (i.e. $V, U \sim x^s$) the reduction procedure is invariant after a number finite of steps. Namely, the laws of evolution become the same at different scales of perceptions for algebraic interaction (aside the renormalized parameters). For instance, consider the elementary operator

$$\mathcal{E}_{ij}^l \circ = \{(x_i - x_j)^l, \circ\} \tag{32}$$

which satisfies

$$\mathcal{E}_{ij}^0 = 0. \tag{33}$$

Moreover, consider the zero-level interaction operator ($K = 1$)

$$\mathcal{L}_U^{(0)} = \sum_{ij}^{N^{(0)}} \mathcal{E}_{ij}^s \tag{34}$$

with algebraic interaction of order s. Since the reduction procedure raises the index of $\mathcal{E}^l$, i.e. $l \to l - 1$, and duplicates the number of elementary operators, the first reduction gives

$$\mathcal{L}_U^{(0)} \longrightarrow \sum_{ij}^{N^{(1)}} \mathcal{E}_{ij}^s + \gamma \mathcal{E}_{ij}^{s-1} \mathcal{E}_{ij}^{s-1} \tag{35}$$

corresponding to the operator $\mathcal{L}_U^{(1)}$. We remark, from (33), that it becomes form-invariant when $s = 1$. A second reduction procedure gives

$$\mathcal{L}_U^{(1)} \longrightarrow \sum_{ij}^{N^{(2)}} \mathcal{E}_{ij}^s + \gamma' \mathcal{E}_{ij}^{s-1} \mathcal{E}_{ij}^{s-1} + \gamma'' (\mathcal{E}_{ij}^{s-2} \mathcal{E}_{ij}^{s-1})^2 + \gamma'' (\mathcal{E}_{ij}^{s-1} \mathcal{E}_{ij}^{s-2})^2. \tag{36}$$

So, from (33), for $s = 2$ no change exists in the form of the evolution operator. This is a general property: for a product of elementary Liouville operators, the reduction procedure operates like

$$\dots \mathcal{E}^l \mathcal{E}^m \dots \longrightarrow \dots \mathcal{E}^l \mathcal{E}^m \dots + (\dots \mathcal{E}^{l-1} \mathcal{E}^m \dots)^2 + (\dots \mathcal{E}^l \mathcal{E}^{m-1} \dots)^2 + \dots \tag{37}$$

where we have omitted the subindex i, j by simplicity. Clearly, from (33) and the above expression, the reduction procedure saturates for a finite number of steps.

To close this section, we remark that the reduction procedure must be stopped when there is no loss memory effect, and then this process cannot be continuously carried out. For instance, this is the case for a system with a finite number of constituents. Also, we noticed that geometric aspects must be considered at every reduction.

4. Wigner function and the classical limit

Since (21) is also valid for quantum system making the appropriate changes, it is instructive to study the semiclassical limit using the Wigner function. In fact, we shall find that the semiclassical limit needs some new conditions because the decoherence term is related to the parameter γ.

The Wigner function ρ_w defined by the Fourier transform of the density operator in coordinate representation is given by

$$\rho_w(x, p, t) = \frac{1}{h} \int d\eta \, e^{ip\eta/\hbar} \rho(x - \eta/2, x + \eta/2, t). \tag{38}$$

Where $\rho(x, y, t)$ is the statistical operator in coordinate representation. Then, from equation (21) and keeping by simplicity only the external potential term, the evolution for the Wigner function becomes

$$\partial_t \rho_w = \{H_S, \rho_w\} + \gamma \left\{ \frac{\partial V}{\partial x}, \left\{ \frac{\partial V}{\partial x}, \rho_w \right\} \right\}$$
$$+ (\hbar^2/24) \left(\frac{\partial^3 V}{\partial x^3} \frac{\partial^3 \rho_w}{\partial p^3} - 2\gamma \frac{\partial^2 V}{\partial x^2} \frac{\partial^4 V}{\partial x^4} \frac{\partial^4 \rho_w}{\partial p^4} \right) + O(\hbar^4). \tag{39}$$

The first two terms are the classical operators (22), (23) and the others are related to the quantum contribution. So, the mesoscopic term related to the parameter γ gives new quantum corrections. The usual semiclassical approximation, when $\gamma = 0$, is the well known relationship

$$\left| \frac{\partial V}{\partial x} \frac{\partial \rho_w}{\partial p} \right| \gg \hbar^2 \left| \frac{\partial^3 V}{\partial x^3} \frac{\partial^3 \rho_w}{\partial p^3} \right| \tag{40}$$

and related roughly to the optical geometrical limit. It must be noted, however, that a similar condition related to the mesoscopic term from (39) condition is ($\gamma \neq 0$)

$$\left| \left(\frac{\partial^2 V}{\partial x^2} \right)^2 \frac{\partial^2 \rho_w}{\partial p^2} \right| \gg \hbar^2 \left| \frac{\partial^2 V}{\partial x^2} \frac{\partial^4 V}{\partial x^4} \frac{\partial^4 \rho_w}{\partial p^4} \right|. \tag{41}$$

Namely, it explores even derivatives in the potential.

Finally, we note that the deduction of the evolution equation (39), for the Wigner function, was obtained assuming (integration by part)

$$i\hbar \partial_x \rho(x - \eta, x + \eta) \, e^{ip\eta/\hbar} \big|_{\eta=0}^{\eta=\infty} = 0 \tag{42}$$

namely, the vanishing of the correlation term at infinity. This requirement is not always verified, for instance, consider the states $\rho = \psi(y)^* \psi(x)$ where the wavefunction is $\psi = \sin kx$ which does not satisfy (42). Nevertheless, the contribution due to the decoherence in (21), produces a fast annulment of the off-diagonal terms in the statistical operator for short-range external potential. In this way, condition (42) can be satisfied for times greater than the decoherence time, and given a solid support to the evolution equation (39) for the Wigner function. Explicitly, consider equations (21)–(24) in one dimension and one mesoparticle of mass m in an external field. In coordinate representation (21) becomes ($\hbar = 1$)

$$\partial_t \rho = \frac{i}{2m} (\partial_x^2 \rho - \partial_y^2 \rho) - i(V(x) - V(y))\rho - \gamma (V'(x) - V'(y))^2 \rho. \tag{43}$$

If the external field has finite range (i.e. $V(x) = 0$ if $|x| > a$) then the density matrix can be written $\rho = \psi(x, t)\psi^*(y, t)$ since for $|x - y| > a$ the crossed term $V'(x)V'(y)$ (in (43)) becomes zero. The equation for the amplitude becomes

$$\mathrm{i}\partial_t \psi = -\frac{1}{2m}\partial_x^2 \psi + V\psi - \mathrm{i}\gamma V'^2 \psi. \tag{44}$$

Since the external potential is time independent, consider the solution $\psi = \mathrm{e}^{-\mathrm{i}Et}\phi(x)$ where E is a complex quantity then, from (44), we have

$$\mathrm{Im}\,E = -\gamma \int \mathrm{d}x\, V'^2 |\phi|^2 \tag{45}$$

which is always a non-positive quantity. Therefore, the non-diagonal elements $\rho(x, y)$ (where $|y - x| > a$) go exponentially to zero.

5. Conclusions and discussion

We have considered a hierarchy of master equations describing the evolution, at different scales of perception, for ensembles of mesoparticles. Explicitly, equation (26) gives us formally the evolution operator $\mathcal{L}^{(n+1)}$ from this one at level n (assumption (iii)). Its deduction requires a systematic coordinate change to the centres of mass, defined by some physical constraints, and the elimination of internal degrees of freedom is carried out assuming loss memory effects. This Markovian approximation is not always valid and then, in such a case, the reduction process must be stopped. The first reduction procedure was carried out explicitly for an ensemble of elementary components (21).

It must be noted that assumptions (i)–(iii), of section 1, are the basis where our reduction procedure was developed. Namely, they make it possible for one to obtain a hierarchy of master equations at different scales of perception for cluster of mesoparticles. Some similarities between (i)–(iii) and those used in the architecture of complex synthetic assembles can be found in [18, 19].

On the other hand, some important points related to the deduction of the evolution equation (21) and (26) are as follows.

(a) The first multipolar order expansion, in the interaction term, implies that the asymptotic limit $t \to +\infty$ must be carried out carefully at different scales [28].

(b) Decoherence effects at every discrete scale appear usually related to the reduction technique (section 1). Thus, decoherence at a macroscopic level is due to the internal complexity of every macroparticle. Quantum superposition is turned into statistical mixture.

(c) The search for invariant systems was carried out explicitly for a model composed of interacting oscillators with an anharmonic term. Other algebraic invariant systems were found.

(d) It was assumed that the internal moments, such as d or m in (13) and (16), are random and independent. Obviously this is not easy to prove and we have only to assume that behaviour. The statistical independence, between these random variables, is a simplification related with our calculations.

(e) The deduction of a general equation like (7) was carried out assuming special initial conditions ($\rho(t = 0) = \rho_R^e \rho_S$). In our specific case of section 2, these conditions do not necessarily hold. More explicitly, the internal interaction cannot be switched-up arbitrary. It is an open problem to prove the validity of our procedure here.

(f) In every step ($n \to n + 1$) it was assumed the loss of memory effects, or equivalent, the existence of a hierarchy of timescales. This is a fundamental assumption to carry out the reduction procedure. If there is no loss of memory, the procedure must be stopped.

To explore possible applications of our prescription, we can consider trends such as wavematter, currently studied theoretically as well as experimentally. After all, interacting atoms in external fields could be considered as mesoparticles. Specifically, with laser cooling techniques it becomes possible to cool atoms so that the quantum nature of atomic centre-of-mass motion becomes important [31, 32]. For instance, equation (43) could be used to study the motion of ultracold sodium atoms exposed to a one-dimensional spatially periodic potential pulsed in time. On the other hand, it can be interesting to note the growing interest in new mechanisms to break Anderson localization in disordered systems [33]. Particularly, there is controversy about the possible enhancement of the localization length for interacting particles [34]. This suggests considering the behaviour of mesoparticles in disordered systems. Namely, an equation like to (21) or (43) with an external random potential. After all, localization is a phenomenon related to coherence which is lost due to internal complexity for mesoparticles. A more detailed treatment of these points, and further physical applications will be given elsewhere.

Acknowledgments

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